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Mathematical Model of the Effect of the Energy on (Y-H-F-X), Types of Fungi, with Exponential Function

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Abstract

The mathematical model depicts the behavior of dichotomous branching, limb anastomosis, lateral branching, and limb death due to crowding, as well as energy consumption. Although, there is an error rate, mathematical modeling reduces the amount of work, time, and money required to obtain the desired result. In this work, we will look at a branched mathematical model based on solving the system of partial equation (PDEs). The results of this solution, will describe the success or failure of the fungi species tested in terms of growth, and we will use some codes in the numerical solution (pplane7, pdepe), since, it was difficult to obtain direct mathematical solutions.

Keywords: Dichotomous Branching, Tip-Hypha Anastomosis, Lateral Branching, Tip Death to Overcrowding, Haphal Death

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نموذج رياضي لتأثير الطاقة على انواع الفطريات (Y-H-F-X) مع الدالة الأسية

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الخلاصة

يصور النموذج الرياضي سلوك التفرع ثنائي التفرع ، ومفاغرة الأطراف ، والتفرع الجانبي ، وموت الأطراف بسبب الازدحام ، وكذلك استهلاك الطاقة. على الرغم من وجود معدل خطأ ، إلا أن النمذجة الرياضية تقلل مقدار العمل والوقت والمال المطلوب للحصول على النتيجة المرجوة. في هذا العمل ، سوف ننظر في نموذج رياضي متفرع يعتمد على حل نظام المعادلات الجزئية (PDEs). ستوضح نتائج هذا الحل نجاح أو فشل أنواع الفطريات المختبرة من حيث النمو ، وقد استخدمنا بعض الأكواد في التحليل العددي (pplane7، pdepe) ، لأنه كان من الصعب الحصول على حلول رياضية مباشرة.

الكلمات المفتاحية: المتفرعة ثنائية التفرع، مفاغرة طرف الخيطية، المتفرعة الجانبية، وفاة الطرف بسبب الزحام، الفطر الميت

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معلومات البحث

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Introduction

Fungi form a large and diverse kingdom of heterotrophic eukaryotes. It is classified within its kingdom known as the kingdom of fungi. These fungi are feathery filaments Because it consists of a mass of fungal filaments closely packed together . Studies show that it includes more than(100) thousand species , these do not contain chlorophyll and get their nutrients from the growth media by the use of enzymes that they secrete. All fungi grow naturally when environmental conditions are available to support growth, such as temperature, humidity, and the presence of food, and their growth continues as long as the environmental conditions continue, and therefore, the fungus continues to carry out all the vital activities that it needs to continue to survive such as breathing, reproduction, development, ..., etc[2].

We developed new models for the growth of fungal mycelia . Partial differential equations reflect the interaction of biomass with the underlying substrate which are the best choice at this size. These models have a complicated mathematical structure including parabolic and hyperbolic elements. As a result, their analytic and numerical features are complex, and a group of any number of types can be expressed during the growth stages of a specific fungus. To make it easier to explain these types, abbreviated symbols for each type are used, like in table (1), which shows several biological types that have been mathematically examined and have been given an explanation and a description of the parameters. We shall blend various species of fungi in this paper.

Table (1): show Branching, Biological Type, Symbol of this Type, and Version.

Biological type	Symbol	Version	Parameter description
Dichotomous branching	Y	$\delta = \alpha_1 n$	α_1 ,is the numbers of tips, produced per unit time per tip.
Tip-hypha anastomosis	**H**	$\delta = -\beta_2 n\rho$	β_2 ,is the rate of tip reconnections per unit, as well as the length of the hypha per unit time.
Lateral branching	**F**	$\delta = \alpha_2 \rho$	α_2 ,is the number, of branches produced per unit, hypha length per unit time.
Tip death due to overcrowding	**X**	$\delta = -\beta_3 \rho^2$	β_3 ,is the rate at which branching is eliminated by overcrowding density limitation.

Reference : [1]

Mathematical Model

We'll look into the new type of the fungal branching with whole vegetarian food consumption. which we will call energy $\Psi(x)$. This energy function is defined as $0 \leq \Psi(x) \leq 1$. If $\Psi(x)=0$. The growth is poor if the fungal does not consume all of the energy. But, if $\Psi(x)=1$, the growths is excellent if the fungi consumes all of the the energy [1,2,3].

We can use the system below to describe hyphal growth:

$$\frac{\partial \rho}{\partial t} = nv - \gamma\rho$$

$$\frac{\partial n}{\partial t} = -\frac{\partial (nv)}{\partial x} + e^{[\delta(p,n)]} - \Psi \quad (1)$$

Where: $\delta(p,n) = \alpha_1 \mathrm{n} - \beta_2 n\rho + \alpha_2\rho - \beta_3\rho^2$

and $\Psi=1$. Then this system (1) becomes[3]:

$$\frac{\partial \rho}{\partial t} = nv - \gamma\rho$$

$$\frac{\partial n}{\partial t} = -\frac{\partial (nv)}{\partial x} + e^{[\alpha_1 \mathrm{n} - \beta_2 n\rho + \alpha_2\rho - \beta_3\rho^2]} - 1 \quad (2)$$

Non-dimensionlision and Stability

To help us in the analysis and numerical solution of the system (2), we'll used the non-dimensional form for the equations . Choosing the references time τ, a references lengths scale $\bar{x}$. And reference scales for hyphal density $\bar{\rho}$, and tip density $\bar{n}$ [4].

$$\rho^* = \frac{\rho}{\bar{\rho}}, \quad n^* = \frac{n}{\bar{n}}, \quad t^* = \frac{1}{\tau}, \quad and\ x^* = \frac{x}{\bar{x}}$$

Substitute in (2), we got:

$$\frac{\partial \rho}{\partial t} = n - \rho$$

$$\frac{\partial n}{\partial t} = -\frac{\partial n}{\partial x} + e^{[\alpha n(1-p)+\beta p(1-p)]} - 1 \qquad (3)$$

Where: $\alpha=\frac{\alpha_1}{\gamma}$ and $\beta=\frac{\alpha_2 v}{\gamma^2}$

Now. to find the steady states when we take from The system of :

n-p=0 → n=p

And we take the other side

$$e^{[\alpha n(1-p)+\beta p(1-p)]} - 1 = 0$$

$$e^{[\alpha n(1-p)+\beta p(1-p)]} = 1$$

$$lne^{[\alpha n(1-p)+\beta p(1-p)]} = ln1$$

$$\alpha n(1-p) + \beta p(1-p) = 0$$

The steady stat clearly that (ρ,n)=(0,0) and (ρ,n)=(1,1). As a result, we take the Jacobain of these equations.

$$J_{(p,n)} = \begin{bmatrix} -1 & 1 \\ -\alpha n + \beta(1-2p) & \alpha(1-p) \end{bmatrix},$$

Now, determent the eigenvalues as λ_i ;i=1,2

Then, it is clear that the steady state is saddle point at (0,0) and stable spiral at (1,1) for all $\alpha, \beta > 0$ and $\beta > \alpha$

see Fig (1) using MATLAP pplane7 [5].

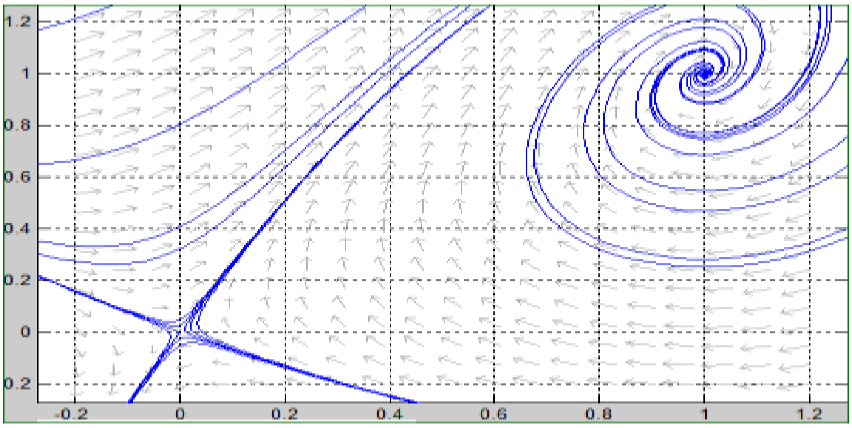

Figure (1): The (n,p) Plane we Notice that, the Trajectory is Saddle Point at (0,0). and Stable Spiral at (1,1), for all α=1 and β=3

Traveling wave solution

We'll talk about the travel wave solution (c). And we will assume that: p(x,t) =P(z), and n(x,t)=N(z), where z=x-ct, P(z) profiled density . and propagation rate (c) of the colony's edge. P(z) is a positive function of z, and N(z) is not a negative function of z. The functions p(x,t), and n(x,t) are moving waves that move in a positive x directional at a constant speed (c), where $c > 0$, Ψ=1 and α=1, β=3 to find the travelling waves solution (c) to the equation in x and t, of the system's (3).

$$\frac{\partial p}{\partial t} = -c\frac{dP}{dz}$$

$$\frac{\partial n}{\partial t} = -c\frac{dN}{dz}$$
$$\frac{\partial n}{\partial x} = \frac{dN}{dz}$$

Therefore , the above equation becomes [6,7]:

$$\frac{dP}{dz} = n - \rho$$

$$\frac{dN}{dz} = -\frac{dN}{dz} + e^{[\alpha n(1-p)+\beta p(1-p)]}, c \neq 1, \quad -\infty < z > \infty \qquad (4)$$

To get the steady state of the above system (4). For all $c < 0$ and $\beta > \alpha$ we'll get (N,P)=(0,0) which is saddle point, and (N,P)=(1,1) is stable spiral, as shown in Fig (2) below:

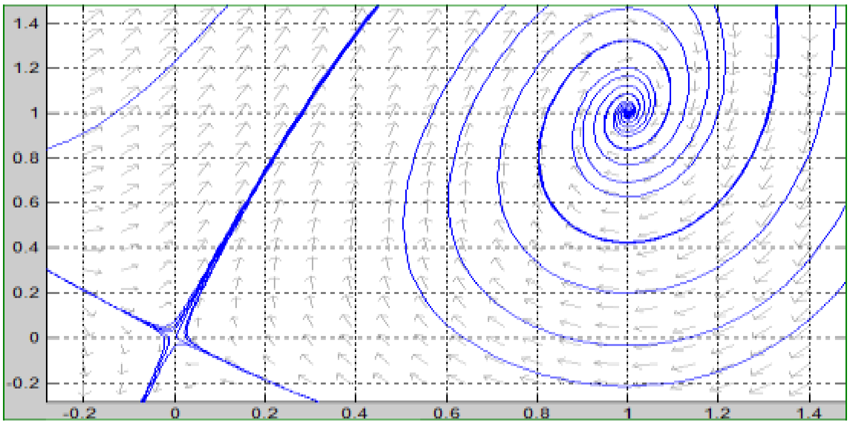

Figure (2) : The (N,P) Plane note That, the Trajectory connect from Saddle Point at (0,0), to stable Spiral at (1,1), for all c=-1 and α=1 , β=3.

Numerical Solution

Now, we'll solve the system (3) in MATLAP using the pdepe code, which clearly shows that the initial condition starts from 1 to 0 (1→ 0) for p and n. Figure (3) shows the behavior for p and n, which clearly shows that the travelling wave is regular over time. And in Figs. (4) and (5), the blue line represents tip of (n), the red line represents branche (p), and Fig. (6), the blue line represents tips (n), the red line represented branches (p), for all α =0.4 and β =0.6, and c=2.3145 ,for time t=1,10,20,....,300 [8,9,10].

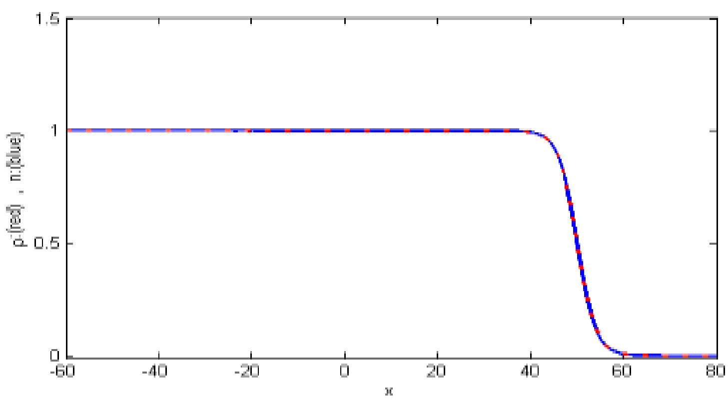

Figure (3): The initial conditional of the system's solution (3), with a parameters α=0.4, β=0.6.

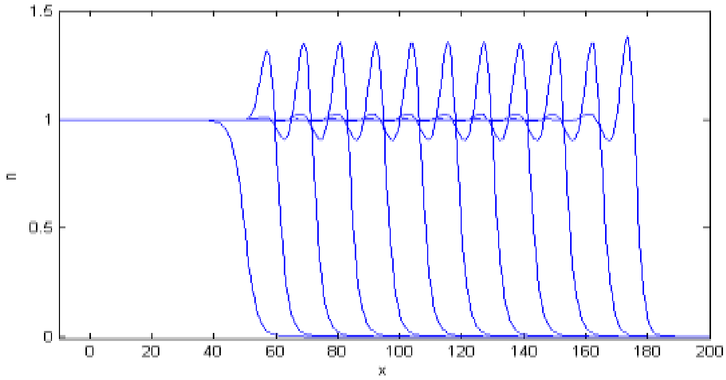

Figure (4): The tips were represented by a blue line (n).

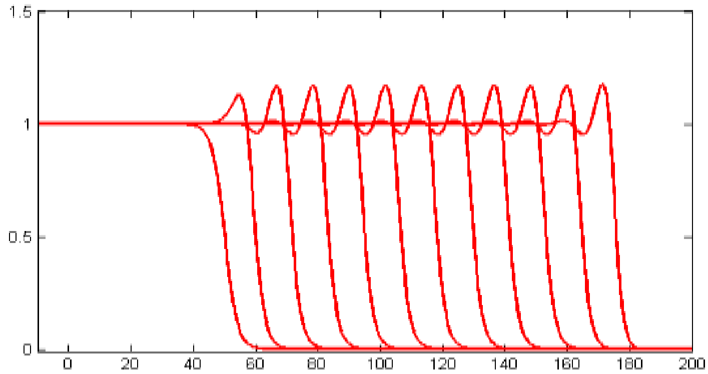

Figure (5): The Branches were Represented by the Red Line (p)

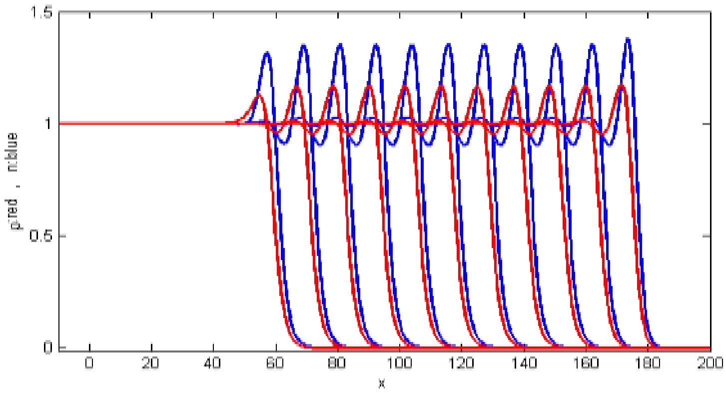

Figure (6): The Blue Line denoted the tips (n), while the Red line denoted the Branches (p).

We will got a relationship between the traveling waves solution (c), and values of α from this operation, and we can see in table (2), that is the travelling waves (c) increase as the value of α increase. See Fig (7).

Table (2): The relation between traveling waves solution (c), and parameter α with taking β=v=1.

α	0.5	1	2	3	4	5	6	7	8	9	10
c	2.89	3.78	5.81	7.58	9.53	11.54	13.57	15.59	17.84	19.79	**21.81**

Reference : Results by MATLAB.

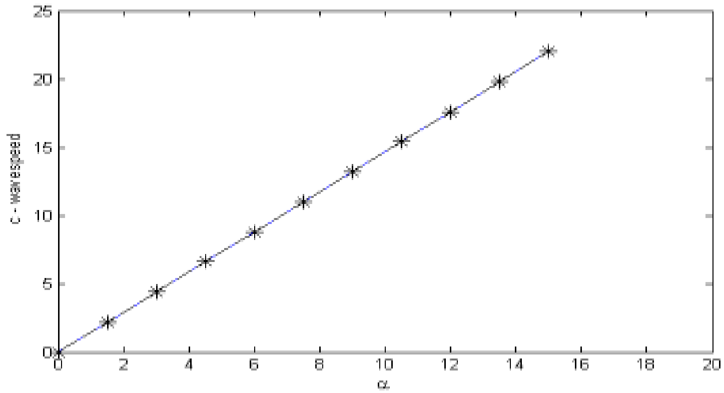

Figure (7): The Relation between Travelling Wave Solution (c), and α.

Now, we are taking the relationships between the travelling waves solution (c), and the parameter β, that we note in table (3), the travelling wave increasing when the values of β increase. See Fig (8).

Table (3): The Relation between Traveling wave (c), and Parameter β with taking α=v=1.

β	0.5	1	2	3	4	5	6	7	8	9	10
c	3.44	3.78	4.36	4.57	5.37	5.64	6.24	6.64	7.35	7.61	**8.27**

Reference: Results by MATLAB.

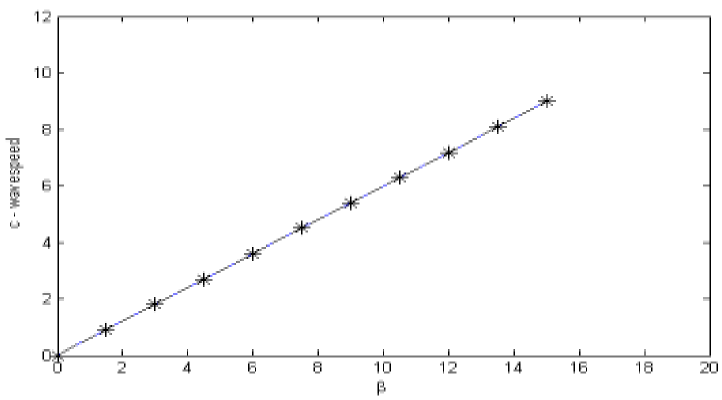

Figure (8): The Relation between Travelling waves Solution (c), and β.

Finally, we are taking the relationship between the travelling waves solution (c), and the values of v, that we note in table (4), the travelling wave increasing when the values of v increase. See Fig (9).

Table (4): The Relation between Traveling Wave c and Parameter v with taking β=α=1.

v	0.5	1	2	3	4	5	6	7	8	9	10
c	3.43	3.78	4.37	4.49	4.63	5.44	5.83	6.34	6.72	7.18	**7.64**

Reference : Results by MATLAB

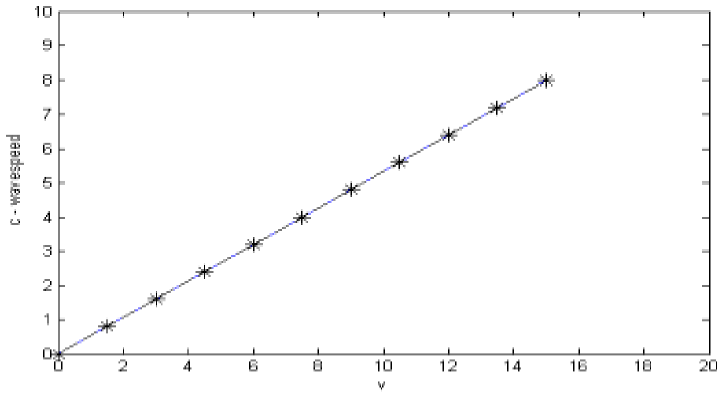

Figure (9) The Relation between Travelling wave Solution c and v.

Conclusion

Based on the results which are shown in Figs (7) and (8); we conclude that the travelling wave solution (c) increases when the values of α, are increasing at the same time, β is still constant.

So, we know the values of $(\alpha = \frac{\alpha_1}{\gamma})$, and notice that it is directly proportional to α_1 ,and inversely proportional γ.

Biologically , this means that the growths increasing whenever α increases , and finally, growth increases as α_1 increases (α_1 , is the number of tips produced per tip, per unit time).

According to Fig. (8); the travelling waves (c) are increasing whenever the values of β, increase at the same time α is still constant.

So, the value of $(\beta = \frac{\alpha_2 v}{\gamma^2})$, is directly proportional to α_2 and v, but inversely proportional to γ^2

Biologically, this means that growth accelerates as the temperature rises. Finally, growth increases as α_2, increases (α_2 ,branches produced per, unit length hypha per unit times). γ, decreasing (γ ,is the rest loss of hyphal (constant for hyphal death)).

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