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Mathematical Model of Tip Death Due to Overcrowding and Tip-tip Anastomosis and Dichotomous Branching

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Abstract

In this research, we have studied the growth status of fungal species when mixing three types of dichotomous branching and Tip death due to overcrowding and Tip-tip anastomosis, this biological phenomenon is represented as a mathematical model as partial differential equations (PDEs). The solution of this system depends on the numerical solution and this solution gives an approximate solution. Some steps in this solution such as steady states, phase plane and travlling wave solution. The results will describe the success or failure of the growth of the types of fungi studied and we used some codes (pplane7, Pdepe) in the numerical solution because there is a kind of difficulty in the direct mathematical solution.

Keywords: Dichotomous branching , Tip death due to overcrowding , Tip-tip anastomosis

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نموذج رياضي للمتفرعة ثنائيه التفرع والموت بسبب الازدحام ومفاغرة طرف طرف

زينب حسين خليل [1] ، أ. د. علي حسين شعاع [2]

الخلاصة

درسنا في هذا البحث حالة نمو الأنواع الفطرية عند خلط ثلاثة أنواع من التفرع ثنائي التفرع ، وموت الأطراف بسبب الازدحام ومفاغرة طرف التلميح ، يتم تمثيل هذه الظاهرة البيولوجية كنموذج رياضي كمعادلات تفاضلية جزئية (PDEs). يعتمد حل هذا النظام على الحل العددي وهذا الحل يعطي حلاً تقريبيًا. بعض الخطوات في هذا الحل مثل الحالات المستقرة ومستوى الطور وموجة العبور. ستصف النتائج نجاح أو فشل نمو أنواع الفطريات المدروسة واستخدمنا بعض الأكواد(pplane7، Pdepe) في التحليل العددي لوجود بعض الصعوبة في الحل الرياضي المباشر.

الكلمات المفتاحية: ثنائية التفرع، مفارغة طرف ، الموت بسبب الاكتظاظ.

انتساب الباحثين

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معلومات البحث
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Introduction

We will speak about a new type of fungal branching with fungal death is Tips death due to overcrowding (X), Tip-tip anastomosis (W) and Dichotomous branching (Y). as shown in Table (1) which illustrates these types[1,2,3].

Table 1: illustrates branching biological type and symbol of this type and versions.

Biological type	Symbol	version
Tip death due to overcrowding	X	$\delta = -\beta_3 p^2$
Tip-tip anastomosis	W	$\delta = -\beta_1 n^2$
Dichotomous branching	Y	$\delta = \alpha_1 n$

Reference:[2,3]

Mathmetical Model

Biological characterization of mushrooms: mathematicians saw that they turn the branches of fungi into letters, and these letters depend on the behavior of the species in terms of [4]

p = hyphen density in unit filament length per unit area.

n = tips density

Fungi also depend on the availability of energy.

The model below represents our goal in this paper

$$\frac{\partial p}{\partial t} = nv - \gamma p$$

$$\frac{\partial n}{\partial t} = -\frac{\partial (nv)}{\partial x} + \delta(p,n) \qquad (1)$$

Where :$\delta(p,n) = -\beta_3 p^2 - \beta_1 n^2 + \alpha_1 n$

Then the system (1) become

$$\frac{\partial p}{\partial t} = nv - \gamma p$$

$$\frac{\partial n}{\partial t} = -\frac{\partial (nv)}{\partial x} - \beta_3 p^2 - \beta_1 n^2 + \alpha_1 n \qquad (2)$$

Non-dimensionlision and Stability

In this part demonstrate how these parameters can be positioned as lower dimensionlision [2,3]

$$\frac{\partial p}{\partial t} = n - p$$

$$\frac{\partial n}{\partial t} = -\frac{\partial n}{\partial x} - \alpha(p^2 + n^2) + \beta n \qquad (3)$$

Whrer $\alpha = \frac{\beta_1 \bar{n}}{\gamma}$ and $\beta = \frac{\alpha_1}{\gamma}$

Now, to find steady states when take from system (3)

$$n - p = 0 \qquad \rightarrow \quad n = p$$

And on the other hand

$$-\alpha(p^2 + n^2) + \beta n = 0 \qquad \rightarrow \quad n = 0 \ \ then \qquad \rightarrow (p,n) = (0,0)$$

$$and \quad n = \frac{\beta}{2\alpha} \qquad \rightarrow \quad p = \frac{\beta}{2\alpha} \qquad \rightarrow (p,n) = \left(\frac{\beta}{2\alpha}, \frac{\beta}{\alpha}\right)$$

So that is clear the steady states are (p,n) = (0,0) and (p,n) = $(\frac{\beta}{2\alpha}, \frac{\beta}{2\alpha})$ threrfore we take Jacobain of these equations [5,6]

$$J(p,n) = \begin{bmatrix} -1 & 1 \\ -2\alpha p & -2\alpha n + \beta \end{bmatrix}$$

We can classify the cirtical points according to the matrix Jacobain (0,0)

$$J(0,0) = \begin{bmatrix} -1 & 1 \\ 0 & \beta \end{bmatrix}$$

Thus $|A - \lambda I| = 0$ we get two values of λ :-

$$\lambda_1 = -1$$

$$\lambda_2 = \beta$$

Then we take the Jacobain at $\left(\frac{\beta}{2\alpha}, \frac{\beta}{2\alpha}\right)$:

$$J(p,n) = \begin{bmatrix} -1 & 1 \\ -\beta & 0 \end{bmatrix}$$

Thus $|A - \lambda I| = 0$ we get two values of λ :-

$$\lambda_{1,2} = \frac{-1 \pm \sqrt{1-4\beta}}{2}$$

We note the probabilities of the β. [7,8,9]

If β is positive, we get the point (0,0) saddle point and the point $\left(\frac{\beta}{2\alpha}, \frac{\beta}{2\alpha}\right)$ stable spiral , as shown in figure (1) .by using (MATLAB pplane7).

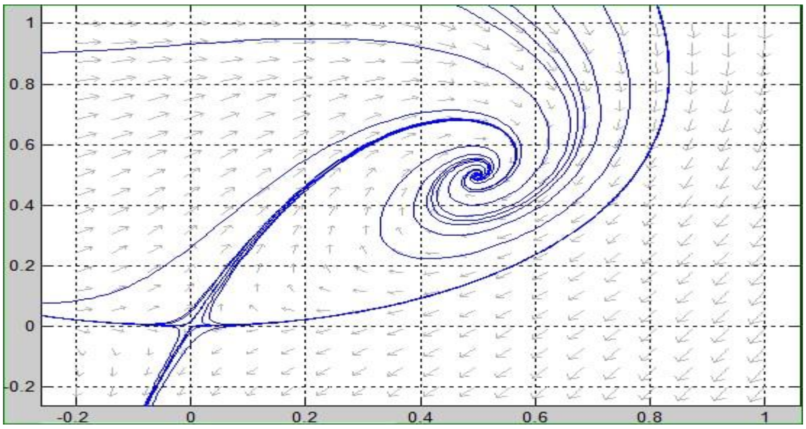

Figure 1 :The (p,n) plane:-note that a trajectory connects the saddle point in (0,0) and stable spiral in point $(\frac{\beta}{2\alpha}, \frac{\beta}{2\alpha})$

Travelling wave solution

In this part, we will speak about the travelling waves solution, let $= x - ct$, and we impose

$$n(x,t) = N(z)$$

$$(4)$$

$$p(x,t) = P(z)$$

where P(z) indicates the density profiles, and (c) rate of propagation of colony. N(z) and P(z) positive function for (z) The functions N(x,t), p(x,t) are traveling and moving at constant speed wave c in positive x − direction , where c > 1 and $\alpha = \beta = 1$. We observe the travelling waves solution of the system in t and t in the form of [10].

$$\frac{\partial p}{\partial t} = -c\frac{\partial P}{\partial z}, \qquad \frac{\partial n}{\partial t} = -c\frac{\partial N}{\partial z}, \qquad \frac{\partial n}{\partial t} = \frac{\partial N}{\partial z}$$

Therefore it becomes the system of

$$\frac{\partial P}{\partial z} = \frac{-1}{c}[N - P]$$

$$\frac{\partial N}{\partial z} = \frac{1}{1-c}[-\alpha(p^2 + n^2) + \beta n], \quad c \neq 0, \quad -\infty < z < \infty \tag{5}$$

We notice the steady states of the system (5) we get the point (p,n)=(0,0) which is saddle point and $(\frac{\beta}{2\alpha}, \frac{\beta}{2\alpha})$. Unstable spiral for c> 1 ,as shown in Figure (2) by using **MATLAB**

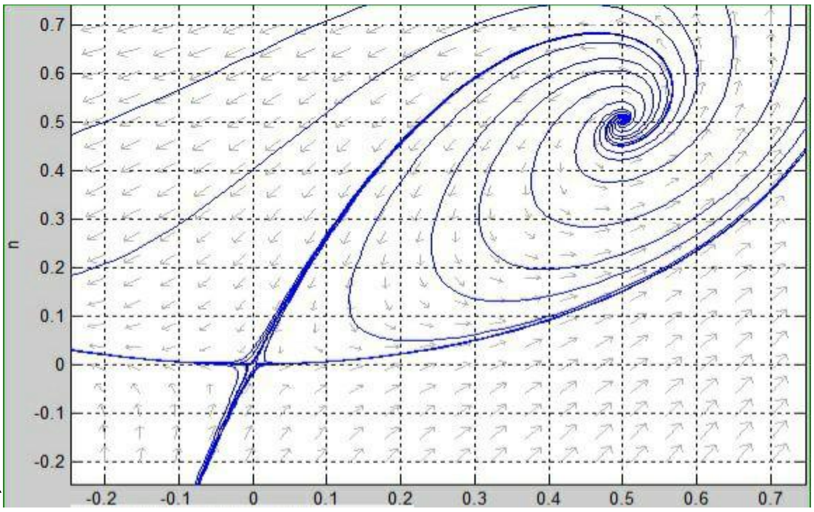

Figure 2: The (P,N) plane note that a trajectory connects when c=2 , $\alpha = \beta = 1$ the saddle point in (0,0) and Unstable spiral in point $(\frac{\beta}{2\alpha}, \frac{\beta}{2\alpha})$

Numercal solution

Since System (2) is completely unsolvable, so we resort to numerical solutions, here we use the (pdepe) code in (MATLAB) to show the behavior of branch and tips.

In Fig (3) shows the solution of System (2) with parameters $\alpha = 0.5$, $\beta = 1$ and c = 3.0657 for time t = 1,10,20,…..400. as shown in figure (3)

In Fig (4) where the blue line represents the tips(n) . as shown in figure (4)

In Fig (5) where the red line represents the branches (p). as shown in figure (5)

In Fig (6) where the blue line represented tips (n) with the red line represented branches (p). as shown in figure (6).

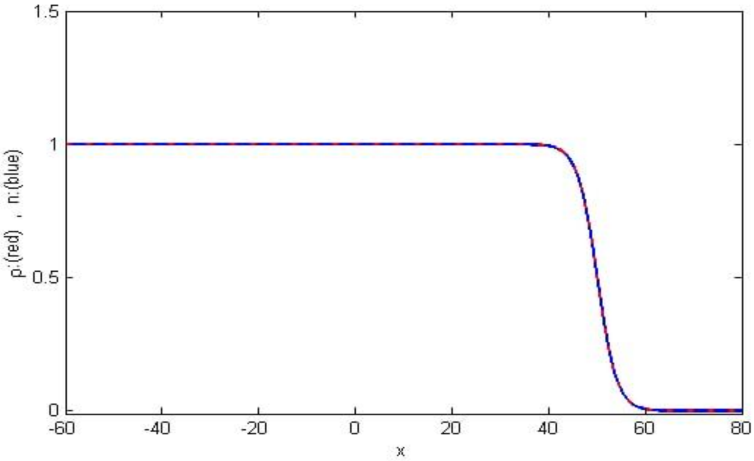

Figure 3: The initial condition of solution to the system (3) with the parameters $\alpha = 0.5$, $\beta = 1$

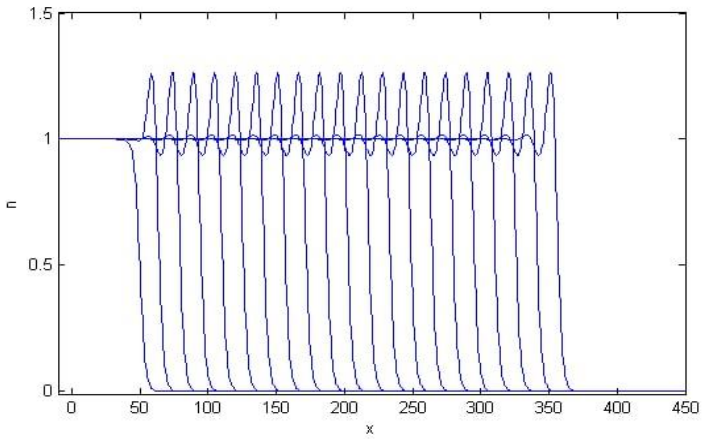

Figure 4:Solution of the system (3) with the parameters $\alpha = 0.5$, $\beta = 1$ and the wave speed c = 3.0657 for t = 1,10,20,………,200 where the blue line represented tips (n).

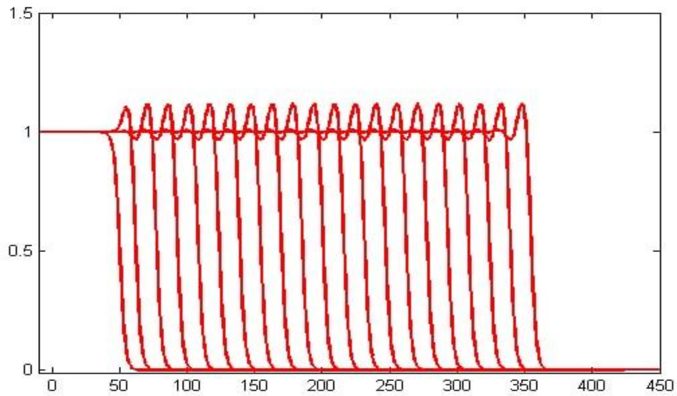

Figure 5 :Solution of the system (3) with the parameters $\alpha = 0.5$, $\beta = 1$ and the wave speed c = 3.0657 for t = 1,10,20,………,200 where the red line represents branches (p).

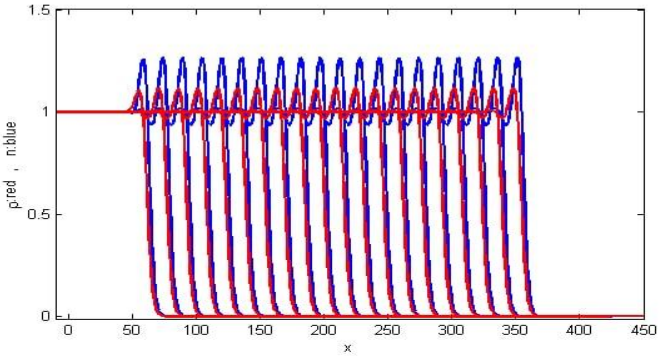

Figure 6 :Solution of the system (3) with the parameters $\alpha = 0.5$, $\beta = 1$ and the wave speed c = 3.0657 for t = 1,10,20,………,200 where the blue line represented tips (n) with the red line represents branches (p).

From these operations, we obtain the relationship between the values of the travelling waves solution c and α taking v $= \beta =$ d = 1, where α increases the travelling waves solution c decreased, as shown in Figure (7)

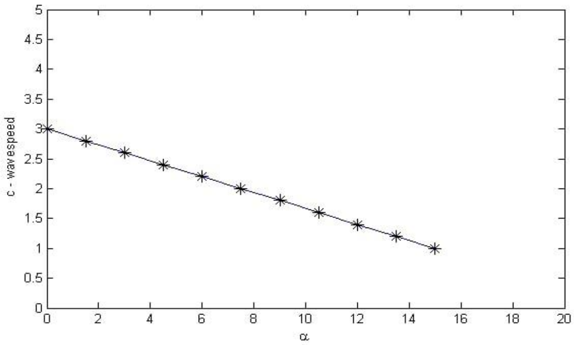

Figure 7 : the relation between waves speed c and α values

Then, we obtain the relationship between the values of the travelling wave solution c and β taking v $= \alpha =$ d = 1, where β increases the travelling waves solution c is increased, as shown in Figure (8)

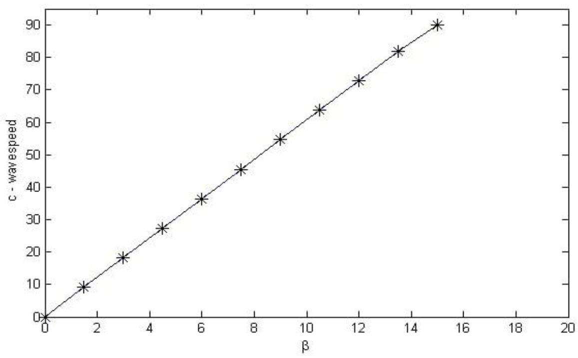

Figure 8 : the relation between waves speed c and β values

Conclusion

In this paper, we recall the most important conclusions drawn from the application of mathematical modeling in the growth of fungi.

1) Mathematical models are able to predict the duration of fungal growth with minimal cost.
2) We used a mathematical solution to shorten the time, cost and effort to get correct results even though there is an error rate.
3) We will take a mathematical model by using the partial solution of the differential system Equations (PDEs) . The results of this solution describe the success or failure of the growth of the fungal species studied.
4) We used some codes in numerical analysis due to some direct difficulties. Mathematical solution.
5) We used non- dimensionlisition, Stability, traveling wave solutions Numerical solutions and numerical solutions to initial value problems by using MATLAB

References

[1] Mich D Alder, 2001, an Introduction to Mathematical Modelling

[2] Ali Hussein Shuaa Al-Taie,2011 ,University of Dundee,Continuum models for fungal growth

[3] Edelstein, L. (1982), the propagation of fungal colonies: A model for tissue growth, Journal of theoretical Biology, J. theor. Biol., 98:679-710.

[4] Michael J. Markowski, (2008), Modeling behavior in vehicular and pedestrian traffic flow

[5] K. and Yamada,(2004) ,Y. Multiple coexistence states for a prey-predator system with cross-diffusion Journal of Differential Equations

[6] . William Boyce and Richard DiPrima, (2005),Elementary Differential Equations and Boundary ValueProblems. John Wiley & Sons Ltd., Hoboken, N.J., eight edition .

[7] Brian Ingalls (2012), Mathematical Modeling in Systems Biology .

[8] Murray J.D. Mathematical Biology II :Spatial Models and biomedical Application. Springer – Verlag New York , Inc., 175 Fifth A venue, New York, NY 10010, USA .(1989)

[9] Schnepf, A. and Roose, T. Modelling the contribution of arbuscular mycorrhizal fungi to plant nutrient uptake. New Phytol, 171:669- 682.(2006)

[10] Shaimardanovich, Durmanov Akmal, and Umarov Sukhrob Rustamovich. "Economic-mathematical modeling of optimization production of agricultural production." Asia Pacific Journal of Research in Business Management 9.6 (2018).

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